ACT 3230 Actuarial Models II

Exam 1 – Chapter 7

February 11, 2009
4:00 p.m. – 5:15 p.m.

Instructor: Sheldon Liu, FSA, FCIA

Exam Notes:

- Each question is worth 5 points for an exam total of 50 points.
- This exam is worth 40% of your final grade.
- Show all of your work. You may ask for another exam booklet if you require one.
- You may use the Illustrative Life Tables for this examination.
- You may use any SOA/CAS-approved calculator. Graphing calculators are not permitted.
- Other books, papers, and electronic devices may not be used for this examination.
- Good luck!

ADDITIONAL NOTES:

- Unless otherwise stated, assume the equivalence principle applies.
- Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. For a fully discrete 10-year deferred whole life insurance of 1000 on (40), you are given:
i) $v = 0.95$
ii) $p_{48} = 0.98077$
iii) $p_{49} = 0.98039$
iv) $A_{50} = 0.35076$

The annual benefit premium of 23.40 is payable during the deferral period. Calculate ${}_8V$, the benefit reserve of this insurance at time $t = 8$, immediately before the premium payment.

2. A 3-year fully discrete endowment insurance policy of 1000 is issued to (x).

You are given:
i) $q_x = 0.1$, $q_{x+1} = 0.2$, $q_{x+2} = 0.3$
ii) i = 0.10
iii) premiums are based on the equivalence principle

Find the reserve at time 1.

3. You are given:
i) $P_x = 0.00646$
ii) $P_{x:\overline{n}|} = 0.04166$
iii) $P^{\;1}_{x:\overline{n}|} = 0.0021$
iv) $P_{x+n} = 0.01426$

Calculate i.

4. Assume mortality follows the Illustrative Life Table and $i = 0.06$. An insurer issues 100 fully discrete whole life policies to independent lives (30) of \$1000 each. Premiums are set using the equivalence principle.

Given that they all survive to duration 10, what is the probability that the aggregate prospective loss at time 10 will exceed 125% of the aggregate reserve?

5. You are given:
i) ${}_t\bar{k}_x = 0.30$
ii) ${}_tE_x = 0.45$
iii) $\bar{A}_{x+t} = 0.52$

Calculate ${}_t\bar{V}(\bar{A}_x)$.

6. You are given:

i) $P_{45:\overline{20|}} = 0.03$

ii) $A^{\,1}_{45:\overline{15|}} = 0.06$

iii) $A_{45:\overline{15|}}^{\quad\;\;1} = 0.40$

iv) $d = 0.054$

Calculate ${}_{15}V_{45:\overline{20|}}$.

.16

7. After taking ACT3230 forty years ago, Stuart Dent was so excited about insurance that he bought a whole life insurance policy that would pay 100,000 at the moment of death. He was 25 at the time, and paid annual premiums at the beginning of each policy year.

Stu is now 65 and retiring, but with the unfortunate downturn in the markets he can no longer afford to pay premiums (despite being a successful actuary)! Instead, he is willing to take a reduced, paid-up whole life policy (still paying at the moment of death). He has not yet paid the premium at age 65.

Determine the appropriate face value of the paid-up policy, assuming mortality follows the Illustrative Life Tables, $i = 0.06$, and a uniform distribution of deaths within each year of age.

8. Assume that mortality follows DeMoivre's Law with $\omega = 100$ and $i = 0.06$.
Find ${}_{10}V(\overline{A}_{50:\overline{25|}})$.

9. (cont'd) Assume that mortality follows DeMoivre's Law with $\omega = 100$ and i = 0.06.
You are also given that ${}_{10}V^{\,1}_{50:\overline{25|}} = 0.040660304$.

Find ${}_{10}V^{(12)}(\overline{A}_{50:\overline{25|}})$.

Notes:

- DeMoivre's Law implies that UDD-eya holds.
- Feel free to use intermediate answers from the previous question.

10. L is the loss-at-issue random variable for a fully continuous whole life insurance of 1 on (30) with net premium determined by the equivalence principle. You are given:

i) $\overline{A}_{50} = 0.7$

ii) ${}^{2}\overline{A}_{30} = 0.3$

iii) $\mathrm{Var}(L) = 0.2$

Calculate ${}_{20}\overline{V}(\overline{A}_{30})$.

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1. For a fully discrete 10-year deferred whole life insurance of 1000 on (40), you are given:
   i) $v = 0.95$
   ii) $p_{48} = 0.98077$
   iii) $p_{49} = 0.98039$
   iv) $A_{50} = 0.35076$

   The annual benefit premium of 23.40 is payable during the deferral period. Calculate ${}_8V$, the benefit reserve of this insurance at time $t = 8$, immediately before the premium payment.

   Solution:

   $${}_8V = \text{APV of future benefit} - \text{APV of future premiums}$$
   $$= 1000\ {}_{2|}A_{48} - 23.40\,\ddot{a}_{48:\overline{2}|}$$
   $$= 1000\ v^2\ {}_2p_{48}\ A_{50} - 23.40(1 + v\,p_{48})$$
   $$= 1000\ (.95)^2(.98077)(0.98039)(.35076) - 23.40[1 + (.95)(.98077)]$$
   $$= 304.3850499 - 45.2025171 = \mathbf{259.1825328}$$

2. A 3-year fully discrete endowment insurance policy of 1000 is issued to (x).

   You are given:
   i) $q_x = 0.1$, $q_{x+1} = 0.2$, $q_{x+2} = 0.3$
   ii) i = 0.10
   iii) premiums are based on the equivalence principle

   Find the reserve at time 1.

   Solution:
   First, find the benefit premium.

   $$1000\,P_{x:\overline{3}|} = 1000\ \frac{A_{x:\overline{3}|}}{\ddot{a}_{x:\overline{3}|}}$$

   $$A_{x:\overline{3}|} = vq_x + v^2\ {}_{1|}q_x + v^3\ {}_{2|}q_x + v^3\ {}_3p_x$$
   $$= 0.1/1.1 + (1-0.1)(0.2)/(1.1)^2 + (1-0.1)(1-0.2)(0.3)/(1.1)^3 + (1-0.1)(1-0.2)(1-0.3)/(1.1)^3$$
   $$= 0.780616078$$

   $$\ddot{a}_{x:\overline{3}|} = 1 + vp_x + v^2\ {}_2p_x$$
   $$= 1 + (1-0.1)/1.1 + (1-0.1)(1-0.2)/(1.1)^2 = 2.41322314$$

Thus, $1000\,P_{x:\overline{3}|} = 1000\,\dfrac{A_{x:\overline{3}|}}{\ddot{a}_{x:\overline{3}|}} = 1000\,(0.780616078/2.41322314) = 323.4744707$

$${}_1L = \begin{cases} 1000v - 323.47 = 585.6164384 & K(x+1) = 0, \text{ prob. } q_{x+1} = 0.2 \\ 1000v^2 - 323.47\,\ddot{a}_{\overline{2}|} = 1000/(1.1)^2 - 323.47(1 + 1/1.1) = 208.9041096 & K(x+1) \geq 1, \text{ prob. } p_{x+1} = 0.8 \end{cases}$$

$$\begin{aligned} 1000\,{}_1V_{x:\overline{3}|} &= E[{}_1L | K(x) \geq 1] \\ &= (585.6164384)(0.2) + (208.9041096)(0.8) \\ &= \mathbf{284.246575} \end{aligned}$$

Alternatively,

$${}_1V_{x:\overline{3}|} = 1 - \frac{\ddot{a}_{x+1:\overline{2}|}}{\ddot{a}_{x:\overline{3}|}}$$

$$\begin{aligned} \ddot{a}_{x:\overline{3}|} &= 1 + vp_x + v^2\,{}_2p_x \\ &= 1 + (1-0.1)/1.1 + (1-0.1)(1-0.2)/(1.1)^2 \\ &= 2.41322314 \end{aligned}$$

$$\ddot{a}_{x+1:\overline{2}|} = 1 + vp_{x+1} = 1 + (1-0.2)/1.1 = 1.727272727$$

$${}_1V_{x:\overline{3}|} = 1 - \frac{\ddot{a}_{x+1:\overline{2}|}}{\ddot{a}_{x:\overline{3}|}} = 1 - (1.727272727 / 2.41322314) = 0.284246575$$

$$1000\,{}_1V_{x:\overline{3}|} = 1000(0.284246575) = \mathbf{284.246575}$$

3. You are given:
   i) $P_x = 0.00646$
   ii) $P_{x:\overline{n}|} = 0.04166$
   iii) $P_{\substack{1\\x:\overline{n}|}} = 0.0021$
   iv) $P_{x+n} = 0.01426$

   Calculate i.

Solution:
Using the "3-premium" idea:

$${}_nV_x = \frac{P_x - P_{\substack{1\\x:\overline{n}|}}}{P_{x:\overline{n}|}^{\ \ 1}} = \frac{P_x - P_{\substack{1\\x:\overline{n}|}}}{P_{x:\overline{n}|} - P_{\substack{1\\x:\overline{n}|}}} = \frac{0.00646 - 0.00211}{0.04166 - 0.00211} = 0.110$$

$${}_nV_x = \frac{P_{x+n} - P_x}{P_{x+n} + d} = \frac{0.01426 - 0.00646}{0.01426 + d} = 0.110 \rightarrow d = 0.0566$$

$$i = \frac{d}{1-d} = \mathbf{0.06}$$

4. Assume mortality follows the Illustrative Life Table and $i = 0.06$. An insurer issues 100 fully discrete whole life policies to independent lives (30) of $1000 each. Premiums are set using the equivalence principle.

   Given that they all survive to duration 10, using normal approximation, what is the probability that the aggregate prospective loss at time 10 will exceed 125% of the aggregate reserve?

   Solution:
   First, find the benefit premium for each one of these policies:

   $$1000P_{30} = 1000\frac{A_{30}}{\ddot{a}_{30}} = 102.48 / 15.8561 = 6.463127755$$

   Then, find the expectation and variance of the prospective loss at time 10 for a single policy:

   $$1000\ {}_{10}V_{30} = 1000A_{40} - 1000P_{30}(\ddot{a}_{40}) = 161.32 - 6.463127755(14.8166) = 65.5584213$$

   $$\begin{aligned} Var({}_{10}L|K(30)\geq 10) &= \left(1000 + \frac{1000P_{30}}{d}\right)^2 [{}^2A_{40} - (A_{40})^2] \\ &= [1000 + 6.4631*(1.06/.06)]^2\ [.04863 - (.16132)^2] \\ &= (1{,}114.181924)^2\ (.022605858) \\ &= 28{,}062.94235 \end{aligned}$$

   On 100 independent variables, let aggregate loss $= S = 100L$

   $E[S] = 100E[{}_{10}L|K(30)\geq 10] = 100(65.5584) = 6{,}555.84$

   $Var(S) = 100Var({}_{10}L|K(30)\geq 10) = 100(28{,}062.94) = 2{,}806{,}294 \rightarrow \sigma = \sqrt{Var(S)} = 1{,}675.19976$

   $$\begin{aligned} \Pr[S > 125\%(E[S])] &= \Pr\left(\frac{S - E[S]}{\sqrt{Var(S)}} > \frac{1.25(6{,}555.84) - 6{,}555.84}{1{,}675.20}\right) \\ &= \Pr[N(0,1) > 0.97836722] = 1 - \Phi(.978) \sim= 1 - .8365 = \mathbf{0.1635} \end{aligned}$$

5. You are given:
   i) ${}_t\bar{k}_x = 0.30$
   ii) ${}_tE_x = 0.45$
   iii) $\bar{A}_{x+t} = 0.52$

   Calculate ${}_t\bar{V}(\bar{A}_x)$.

   Solution:
   Being given ${}_t\bar{k}_x$ should lead you to consider the retrospective formula: ${}_t\bar{V}(\bar{A}_x) = \bar{P}(\bar{A}_x)\ \bar{s}_{x:\overline{t}|} - {}_t\bar{k}_x$

   But, there is no obvious way to get $\bar{P}(\bar{A}_x)$ or $\bar{s}_{x:\overline{t}|}$.

   We know that ${}_t\bar{V}(\bar{A}_x) = \dfrac{\bar{A}_{x+t} - \bar{A}_x}{1 - \bar{A}_x}$ and are given $\bar{A}_{x+t}$, so need $\bar{A}_x$.

   $${}_t\bar{k}_x = \frac{\bar{A}^{1}_{x:\overline{t}|}}{{}_tE_x} \rightarrow \bar{A}^{1}_{x:\overline{t}|} = ({}_t\bar{k}_x)({}_tE_x) = (0.30)(0.45) = 0.135$$

$$\overline{A}_x = \overline{A}^{\,1}_{x:\overline{t|}} + {}_tE_x(\overline{A}_{x+t}) = 0.135 + (0.45)(0.52) = 0.369$$

$$_tV(\overline{A}_x) = \frac{\overline{A}_{x+t} - \overline{A}_x}{1-\overline{A}_x} = \frac{.52-.369}{1-.369} = \mathbf{0.239302694}$$

6. You are given:
   i) $P_{45:\overline{20|}} = 0.03$
   ii) $A^{\,1}_{45:\overline{15|}} = 0.06$
   iii) $A_{45:\overline{15|}}^{\;\;\;\;1} = 0.40$
   iv) $d = 0.054$

   Calculate ${}_{15}V_{45:\overline{20|}}$.

   Solution:

   $${}_{15}k_{45} = A^{\,1}_{45:\overline{15|}} \,/\, {}_{15}E_{45} = 0.06 / 0.4 = 0.15$$
   $$A_{45:\overline{15|}} = A^{\,1}_{45:\overline{15|}} + A_{45:\overline{15|}}^{\;\;\;\;1} = 0.06 + 0.4 = 0.46$$
   $$\ddot{a}_{45:\overline{15|}} = (1 - A_{45:\overline{15|}}) / d = (1-0.46)/0.054 = 10$$
   $$\ddot{s}_{45:\overline{15|}} = \ddot{a}_{45:\overline{15|}} \,/\, {}_{15}E_{45} = 10 / 0.4 = 25$$

   $${}_{15}V_{45:\overline{20|}} = P_{45:\overline{20|}}\,\ddot{s}_{45:\overline{15|}} - {}_{15}k_{45} = 0.03(25) - 0.15 = \mathbf{0.60}$$

7. After taking ACT3230 forty years ago, Stuart Dent was so excited about insurance that he bought a whole life insurance policy that would pay 100,000 at the moment of death. He was 25 at the time, and paid annual premiums at the beginning of each policy year.

   Stu is now 65 and retiring, but with the unfortunate downturn in the markets he can no longer afford to pay premiums (despite being a successful actuary)! Instead, he is willing to take a reduced, paid-up whole life policy (still paying at the moment of death). He has not yet paid the premium at age 65.

   Determine the appropriate face value of the paid-up policy, assuming mortality follows the Illustrative Life Tables, $i = 0.06$, and a uniform distribution of deaths within each year of age.

   Solution:
   A paid-up policy means that there are no more future premiums. Therefore, the value of the reserve at age 65 is equal to the present value of future benefits, noting that the future benefits must be reduced if no more premiums are expected.

   Let F = the new face value of the paid-up policy (i.e. the new benefit amount)
   $$100{,}000\;{}_{40}V(\overline{A}_{25}) = F\,\overline{A}_{65}$$

$$ {}_{40}V(\bar{A}_{25}) = \bar{A}_{65} - P(\bar{A}_{25})\ddot{a}_{65} = \left[1 - P(\bar{A}_{25})\frac{\ddot{a}_{65}}{\bar{A}_{65}}\right]\bar{A}_{65} = \left[1 - \frac{P(\bar{A}_{25})}{P(\bar{A}_{65})}\right]\bar{A}_{65} $$

Assuming UDD-eya,

$P(\bar{A}_x) = \frac{i}{\delta}P_x$ and $\bar{A}_x = \frac{i}{\delta}A_x$

So, ${}_{40}V(\bar{A}_{25}) = \left[1 - \frac{P_{25}}{P_{65}}\right]\bar{A}_{65}$

Using the ILT,

$A_{25} = 0.08165,\quad A_{65} = 0.43980,\quad \ddot{a}_{25} = 16.2242,\quad \ddot{a}_{65} = 9.8969$

$P_{25} = A_{25} / \ddot{a}_{25} = 0.08165 / 16.22418333 = 0.005032611$

$P_{65} = A_{65} / \ddot{a}_{65} = 0.43980 / 9.896866667 = 0.044438307$

$F = 100{,}000\left[1 - \frac{P_{25}}{P_{65}}\right] = 100{,}000\ (1 - 0.005032611 / 0.044438307) =$ **88,675.06**

Note that solving for the value of the reserve was unnecessary, but if you had to:

$\bar{A}_{65} = \frac{i}{\delta}A_{65} = (.06)/\ln(1.06) * (0.43980) = 0.452865874$

$100{,}000\ {}_{40}V(\bar{A}_{25}) = 88{,}675(.452865874) = 40{,}157.91$

8. Assume that mortality follows DeMoivre's Law with $\omega = 100$ and $i = 0.06$.
Find ${}_{10}V(\bar{A}_{50:\overline{25|}})$.

Solution:

$$ {}_{10}V(\bar{A}_{50:\overline{25|}}) = \bar{A}_{60:\overline{15|}} - P(\bar{A}_{50:\overline{25|}})\ddot{a}_{60:\overline{15|}} $$

$$ \bar{A}_{60:\overline{15|}} = \bar{A}^{1}_{60:\overline{15|}} + A_{60:\overline{15|}}^{\ \ \ \ 1} = \frac{\bar{a}_{\overline{15|}}}{100-60} + v^{15}\left(\frac{100-60-15}{100-60}\right) = \left(\frac{1}{40}\right)\left(\frac{1-v^{15}}{\delta}\right) + v^{15}\left(\frac{100-60-15}{100-60}\right) $$

$$ = 0.250019675 + 0.260790663 = 0.510810338 $$

$$ A_{60:\overline{15|}} = A^{1}_{60:\overline{15|}} + A_{60:\overline{15|}}^{\ \ \ \ 1} = \frac{a_{\overline{15|}}}{100-60} + v^{15}\left(\frac{100-60-15}{100-60}\right) = \left(\frac{1}{40}\right)\left(\frac{1-v^{15}}{i}\right) + v^{15}\left(\frac{100-60-15}{100-60}\right) $$

$$ = 0.242806225 + 0.260790663 = 0.503596888 $$

$$ \ddot{a}_{60:\overline{15|}} = \left(\frac{1 - A_{60:\overline{15|}}}{d}\right) = (1 - 0.53596888)(1.06/.06) = 8.769788313 $$

$$ \bar{A}_{50:\overline{25|}} = \bar{A}^{1}_{50:\overline{25|}} + A_{50:\overline{25|}}^{\ \ \ \ 1} = \frac{\bar{a}_{\overline{25|}}}{100-50} + v^{25}\left(\frac{100-50-25}{100-50}\right) $$

$$ = 0.263262654 + 0.116499315 = 0.379761969 $$

$$ A_{50:\overline{25|}} = A^{1}_{50:\overline{25|}} + A_{50:\overline{25|}}^{\ \ \ \ 1} = \frac{a_{\overline{25|}}}{100-50} + v^{25}\left(\frac{100-50-25}{100-50}\right) $$

$$ = 0.255667123 + 0.116499315 = 0.372166438 $$

$$\ddot{a}_{50:\overline{25|}} = \left(\frac{1 - A_{50:\overline{25|}}}{d}\right) = (1 - 0.372166438)(1.06/.06) = 11.09172625$$

$$P(\overline{A}_{50:\overline{25|}}) = \overline{A}_{50:\overline{25|}} / \ddot{a}_{50:\overline{25|}} = 0.379761969 / 11.09172625 = 0.034238311$$

$$\begin{aligned} {}_{10}V(\overline{A}_{50:\overline{25|}}) &= \overline{A}_{60:\overline{15|}} - P(\overline{A}_{50:\overline{25|}})\,\ddot{a}_{60:\overline{15|}} = 0.510810338 - (0.034238311)(8.769788313) \\ &= \mathbf{0.210547601} \end{aligned}$$

9. (cont'd) Assume that mortality follows DeMoivre's Law with $\omega = 100$ and $i = 0.06$. You are also given that ${}_{10}V_{\substack{1\\50:\overline{25|}}} = 0.040660304$.

   Find ${}_{10}V^{(12)}(\overline{A}_{50:\overline{25|}})$.

   Notes:
   - DeMoivre's Law implies that UDD-eya holds.
   - Feel free to use intermediate answers from the previous question.

   Solution:

   Under UDD-eya,

   $${}_{10}V^{(12)}(\overline{A}_{50:\overline{25|}}) = {}_{10}V(\overline{A}_{50:\overline{25|}}) + \beta(12) * P^{(12)}(\overline{A}_{50:\overline{25|}})\,{}_{10}V_{\substack{1\\50:\overline{25|}}}$$

   Re: (from previous question)

   $${}_{10}V(\overline{A}_{50:\overline{25|}}) = 0.210547601,$$
   $$\overline{A}_{50:\overline{25|}} = 0.379761969,$$
   $$\ddot{a}_{50:\overline{25|}} = 11.09172625,$$
   $$A_{\substack{\\50:\overline{25|}}}^{\;\;\;1} = 0.116499315$$

   $$\begin{aligned} \ddot{a}^{(12)}_{50:\overline{25|}} &= \alpha(12)\,\ddot{a}_{50:\overline{25|}} - \beta(12)(1 - {}_{25}E_{50}) \\ &= (1.00028)(11.09172625) - (.46812)(1 - 0.116499315) = 10.6812476 \end{aligned}$$

   $$P^{(12)}(\overline{A}_{50:\overline{25|}}) = \overline{A}_{50:\overline{25|}} / \ddot{a}^{(12)}_{50:\overline{25|}} = 0.379761969 / 10.6812476 = 0.035554084$$

   $$\begin{aligned} {}_{10}V^{(12)}(\overline{A}_{50:\overline{25|}}) &= {}_{10}V(\overline{A}_{50:\overline{25|}}) + \beta(12) * P^{(12)}(\overline{A}_{50:\overline{25|}})\,{}_{10}V_{\substack{1\\50:\overline{25|}}} \\ &= 0.210547601 + (.46812) * (0.035554084)(0.040660304) = \mathbf{0.211676733} \end{aligned}$$

   Alternatively,

   $${}_{10}V^{(12)}(\overline{A}_{50:\overline{25|}}) = \overline{A}_{60:\overline{15|}} - P^{(12)}(\overline{A}_{50:\overline{25|}})\,\ddot{a}^{(12)}_{60:\overline{15|}}$$

   Re: (from previous question)

   $\overline{A}_{60:\overline{15|}} = 0.510810338,$ $\qquad$ $\overline{A}_{50:\overline{25|}} = 0.379761969,$

   $\ddot{a}_{60:\overline{15|}} = 8.769788313,$ $\qquad$ $\ddot{a}_{50:\overline{25|}} = 11.09172625,$

$A_{\underset{60:\overline{15|}}{1}} = 0.260790663, \qquad A_{\underset{50:\overline{25|}}{1}} = 0.116499315,$

$$\ddot{a}^{(12)}_{50:\overline{25|}} = \alpha(12)\,\ddot{a}_{50:\overline{25|}} - \beta(12)(1 - {}_{25}E_{50})$$
$$= (1.00028)(11.09172625) - (.46812)(1 - 0.116499315) = 10.6812476$$

$$P^{(12)}(\overline{A}_{50:\overline{25|}}) = \overline{A}_{50:\overline{25|}} \,/\, \ddot{a}^{(12)}_{50:\overline{25|}} = 0.379761969 / 10.6812476 = 0.035554084$$

$$\ddot{a}^{(12)}_{60:\overline{15|}} = \alpha(12)\,\ddot{a}_{60:\overline{15|}} - \beta(12)(1 - {}_{15}E_{60})$$
$$= (1.00028)(8.769788313) - (.46812)(1 - 0.260790663) = 8.426205179$$

$${}_{10}V^{(12)}(\overline{A}_{50:\overline{25|}}) = \overline{A}_{60:\overline{15|}} - P^{(12)}(\overline{A}_{50:\overline{25|}})\,\ddot{a}^{(12)}_{60:\overline{15|}}$$
$$= 0.510810338 - (0.035554084)(8.426205179) = \mathbf{0.211224331}$$

10. L is the loss-at-issue random variable for a fully continuous whole life insurance of 1 on (30) with net premium determined by the equivalence principle. You are given:

i) $\overline{A}_{50} = 0.7$
ii) ${}^2\overline{A}_{30} = 0.3$
iii) $\text{Var}(L) = 0.2$

Calculate ${}_{20}\overline{V}(\overline{A}_{30})$.

Solution:

$$\text{Var}(L) = \frac{{}^2\overline{A}_{30} - (\overline{A}_{30})^2}{(1-\overline{A}_{30})^2} \rightarrow \quad 0.2 = \frac{0.3 - (\overline{A}_{30})^2}{(1-\overline{A}_{30})^2}$$

$$\rightarrow \quad 0.2(1 - 2\overline{A}_{30} + \overline{A}_{30}{}^2) = 0.3 - \overline{A}_{30}{}^2$$
$$1.2\overline{A}_{30}{}^2 - 0.4\overline{A}_{30} - 0.1 = 0$$

Re: Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\overline{A}_{30} = \frac{.4 \pm \sqrt{(.4)^2 - 4(1.2)(-.1)}}{2(1.2)} = \frac{.4 \pm \sqrt{.64}}{2.4} = 0.5 \text{ or } -0.166$$

And since PV of whole life insurance should not be negative, $\overline{A}_{30} = 0.5$.

$${}_{20}\overline{V}(\overline{A}_{30}) = \frac{\overline{A}_{50} - \overline{A}_{30}}{1 - \overline{A}_{30}} = \frac{.7 - .5}{1 - .5} = \mathbf{0.4}$$